Analytical Image Reconstruction Methods for Medical Tomography
- Recent Advances and A New Uniqueness Result -

Hiroyuki Kudo
Graduate School of Systems and Information Engineering, University of Tsukuba, Japan
October 27, 2006 (submitted)

Image reconstruction is a key step in tomographic medical imaging. Roughly speaking, the image reconstruction methods can be classified into analytical methods and iterative (or algebraic) methods. In this presentation, we review advances on the analytical image reconstruction methods since 1990. In particular, we pick up the following two important reconstruction problems. The first topic is the 2-D region of interest (ROI) reconstruction problem in which our aim is to reconstruct an object \( f(x, y) \) on a ROI from its projection data \( p(r, \theta) \) truncated with respect to the radial variable \( r \). The second topic is the cone-beam reconstruction problem in which our aim is to reconstruct a 3-D object \( f(x, y, z) \) from a set of cone-beam projections \( g_{\lambda}(u, v) \) measured by moving an x-ray source along some curve \( \vec{a}(\lambda); \lambda \in \Lambda \) ((\( u, v \)) denotes the planar detector coordinates).

In the 2-D ROI reconstruction problem, we first review the super-short-scan methods developed for fan-beam tomography [1, 2] and the DBP (differentiated backprojection) + Hilbert transform methods to achieve exact ROI reconstruction from truncated projection data [3-5]. These surprising results demonstrated that projection data required for exact reconstruction of a ROI (a small portion of the object) can be significantly reduced from the ordinary complete projection data. Next, we present our recent work in this direction which has not been published yet. This work focuses on how the previous uniqueness results for the ROI reconstruction problem [3-5] can be strengthened when \( a \) priori knowledge on \( f(x, y) \) (in the form that \( f(x, y) \) is known \( a \) priori on some small region) is available. We have proved the following new uniqueness result.

Let \( S \) be a pathwise connected set (non-convex set is allowed) which corresponds to a ROI. Let \( H \) be a subset of \( S \) on which the 1-D Hilbert transform \( \mathbf{H}f(x, y) \) is accessible using the DBP method [3-5]. Then, the object \( f(x, y) \) is uniquely determined on \( S \) if the following two conditions are satisfied.

1. \( f(x, y) \) is known \( a \) priori on the set \( K = S \setminus H \).
2. There exists a subset (possibly a small subset) of \( S \) (denoted by \( B \)) on which both \( \mathbf{H}f(x, y) \) and \( f(x, y) \) are known.

By using \( a \) priori knowledge, this new result allows us to prove the uniqueness of solutions for a wide class of situations in which any previous uniqueness result [3-5] cannot be applied. For example, the interior problem [6] (a famous unsolved problem) is shown to have a unique solution with a tiny \( a \) priori knowledge on \( f(x, y) \).

In the cone-beam reconstruction problem, in spite of a lot of literature on this topic, we limit our attention to the following key concepts. These include the Feldkamp algorithm for a circular source trajectory [7], Tuy’s data sufficiency condition for exact reconstruction [8], Grangeat’s algorithm using the first derivative of 3-D Radon transform [9], cone-beam filtered backprojection (FBP) method [10, 11], and Katsevich’s FBP method [12, 13]. When research on the cone-beam reconstruction began to attract much attention in 1990, the approximate Feldkamp algorithm combined with a circular source trajectory was the only choice for manufacturers to use in practical scan-
ners. Now, however, according to the very active research, it became possible to achieve exact cone-beam reconstruction using a complete source trajectory (such as a spiral trajectory) satisfying Tuy’s data sufficiency condition. We explain how such a progress could be made since 1990. Finally, we note that a very important work which we omit (due to the lack of time) but is surprising is [14].

References


